

TOPIC E

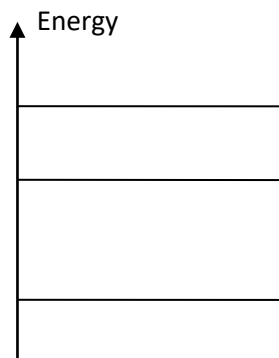
Paper 2 Exam Questions

1.

(a) Explain how the emission lines in the spectrum of a gas provide evidence for discrete energy levels within atoms.

[2]

The diagram shows three energy levels of a hypothetical gas.



Transitions between these three levels give rise to photons of three different wavelengths. Two of these wavelengths are 481 nm and 652 nm. The third transition gives rise to an ultraviolet photon.

(b) Draw arrows to identify the transitions that give rise to the wavelengths of 481 nm and 652 nm.

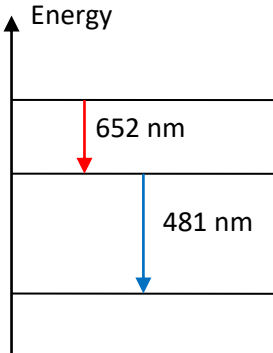
[2]

(c) Calculate the wavelength of the ultraviolet photon.

[3]

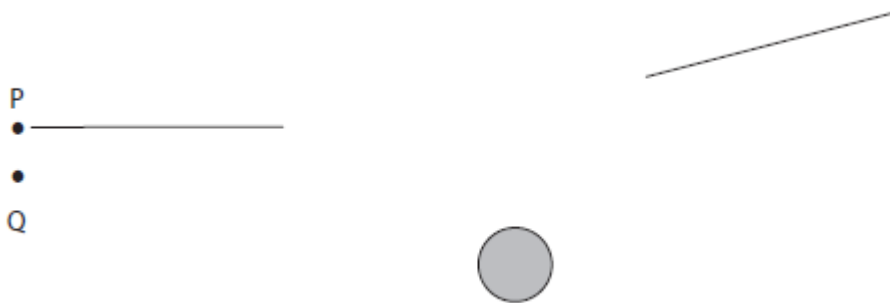
(d) The emission spectrum of hydrogen contains a red spectral line of wavelength 656 nm which comes from a transition from the level $n = 3$ to the level $n = 2$. White light is transmitted through hydrogen gas at room temperature. The absorption spectrum does **not** contain a line at 656 nm. Explain this observation.

[2]

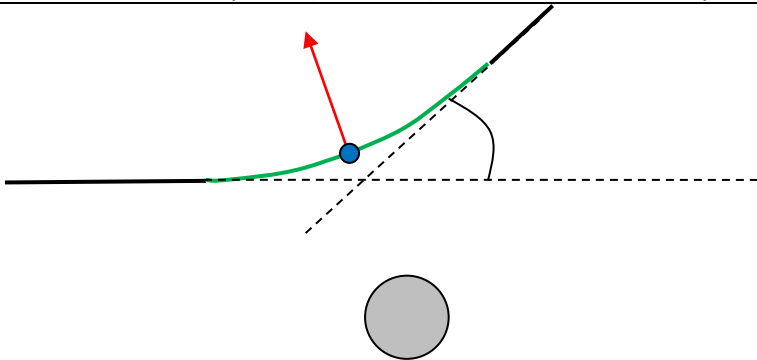
| Question 1 | | Answers | Marks |
|------------|---|---|-------|
| a | | <p>The emitted light does not have a continuous spectrum but has specific, discrete wavelengths✓</p> <p>Since a photon of light has energy hf the atom must have specific, discrete energies✓</p> | 2 |
| b | |  <p>Red arrow✓</p> <p>Blue arrow✓</p> | 2 |
| c | i | <p>Energy of red photon $E = \frac{hc}{\lambda} = \frac{1.24 \times 10^{-6}}{652 \times 10^{-9}} = 1.9018 \text{ eV}$ and energy of blue photon</p> $E = \frac{hc}{\lambda} = \frac{1.24 \times 10^{-6}}{481 \times 10^{-9}} = 2.5780 \text{ eV} \quad \checkmark$ <p>Energy of UV photon $1.9018 + 2.578 = 4.4798 \text{ eV} \quad \checkmark$</p> $\lambda = \frac{hc}{E} = \frac{1.24 \times 10^{-6}}{4.4798} = 277 \text{ nm} \quad \checkmark$ <p>OR</p> $\frac{hc}{\lambda_{UV}} = \frac{hc}{\lambda_1} + \frac{hc}{\lambda_2} \quad \checkmark$ $\lambda_{UV} = \frac{\lambda_1 \lambda_2}{\lambda_1 + \lambda_2} \quad \checkmark$ $\lambda_{UV} = \frac{481 \times 652}{481 + 652} = 277 \text{ nm} \quad \checkmark$ | 3 |
| d | | <p>There will be an absorption line if there are electrons in the state $n = 2$ so they can absorb a photon and go to $n = 3$✓</p> <p>But at a low temperature almost all electrons are in the state $n = 1$ so there can be no absorption✓</p> | 2 |

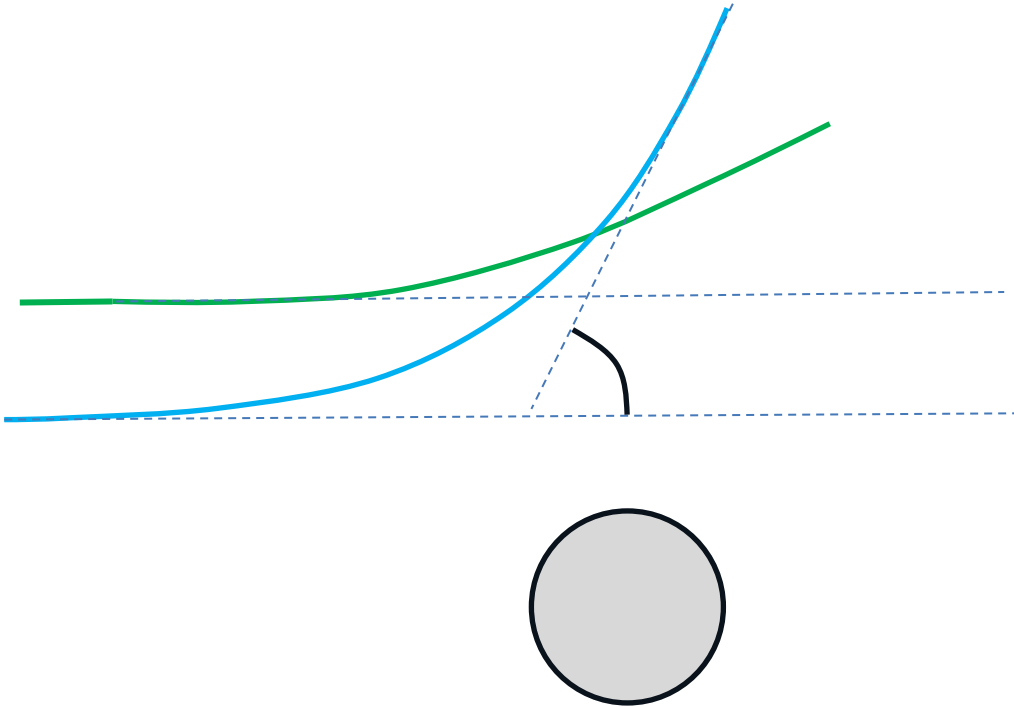
2.

- (a) Explain why in their experiment Geiger and Marsden used:
- (i) an evacuated enclosure [1]
 - (ii) a gold foil that was very thin [1]
 - (iii) a beam of alpha particles that was very narrow. [1]
- (b) State the name of the force responsible for the deflection of the alpha particles. [1]
- (c) (i) Describe the deflections of the alpha particles by the gold foil. [2]
(ii) Outline how the results of this experiment led to the Rutherford model of the atom. [3]
- (d) The diagram shows a partially completed path of an alpha particle that left point P as it scatters past a nucleus of gold.



- On a copy of the diagram:
- (i) complete the path [1]
 - (ii) draw lines to clearly show the angle of deflection of this alpha particle [2]
 - (iii) draw an arrow to indicate the direction of the force on the alpha particle at the point of closest approach. [1]
- (e) (i) A second alpha particle is shot at the nucleus from position Q with identical kinetic energy, in a direction parallel to that of the alpha particle at P. On your diagram, draw the path of this particle. [2]
(ii) Discuss how, if at all, the answer to (e) (i) would change if the nucleus of gold were replaced by a nucleus of another, heavier, isotope of gold. [2]

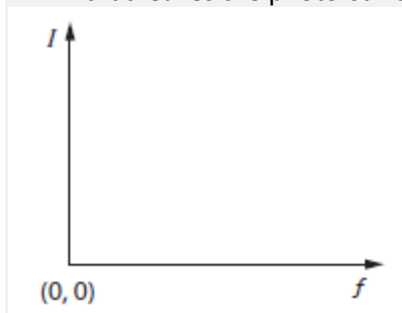
| Question 2 | | Answers | Marks |
|------------|----------------|--|-------|
| a | i | To avoid collisions of alpha particles with air molecules which would have deflected the alphas✓ | 1 |
| a | ii | To avoid absorption of alpha particles as well as avoid multiple scatterings✓ | 1 |
| a | iii | So that the scattering angle could be measured accurately✓ | 1 |
| b | | The electric force✓ | 1 |
| c | i | Most alpha particles went through the foil with no or little deflection✓ Very occasionally, alpha particles were scattered at very large scattering angles✓ | 2 |
| c | ii | The large angle scatterings required a huge electric force✓ The electric force is proportional to $\frac{1}{r^2}$ ✓ So, such a force could be provided if the positive charge of the atom was concentrated in a sphere (10^5 times) smaller than that the prevailing model✓ | 3 |
| d | i ii iii |  <p>Curved path✓ Dotted lines extended from straight paths✓ Direction of force radially away from nucleus✓ Scattering angle marked ✓</p> | 4 |

| | | | |
|---|----|---|---|
| e | i |  <p>Two max from Approaches closer✓ Scattering angle larger✓ Deviation from straight line starts earlier✓</p> | |
| e | ii | <p>The isotope has the same electric charge✓ So, nothing would change (in the approximation that the nucleus does not recoil)✓</p> | 2 |

3.

(a) In a photoelectric effect experiment a constant number of photons is incident on a photo-surface.

- Outline what is meant by *photons*. [2]
- On a copy of the axes below, sketch a graph to show the variation of the electric current I that leaves the photo-surface, with photon frequency f . [2]

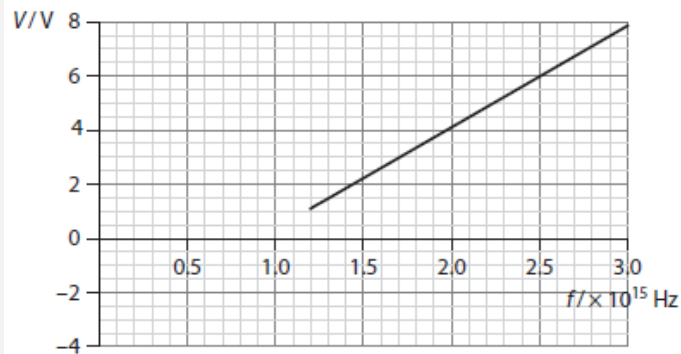


- Explain the features of the graph you drew in (ii). [2]

(b) (i) State one feature of the photoelectric effect that cannot be explained by the wave theory of light. [1]

(ii) Describe how the feature stated in (b) (i) is explained by the photon theory of light. [2]

(c) In another experiment, a source of constant intensity and variable frequency f is incident on a metallic surface. The graph shows the variation of the stopping potential V with photon frequency f , for a particular value of intensity.



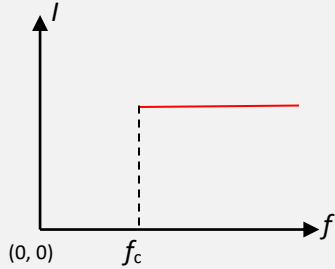
Use the graph to estimate:

(i) the work function of the metallic surface [2]

(ii) the Planck constant obtained from this experiment [3]

(iii) the longest wavelength of light that will result in electron being emitted from the surface. [1]

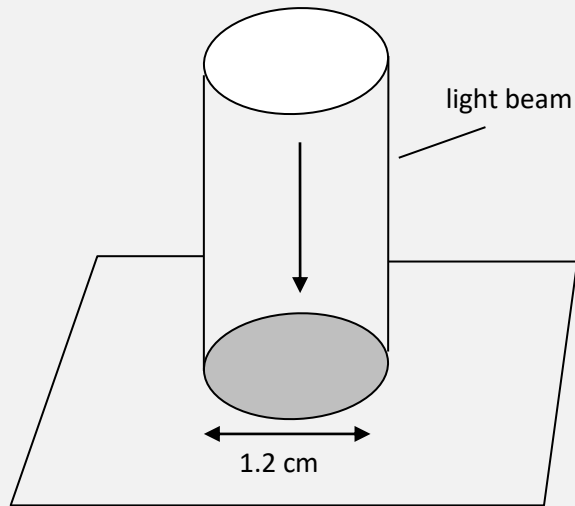
(d) The intensity of the source in (c) is doubled. Discuss how the graph in (c) will change, if at all. [2]

| Question 3 | | Answers | Marks |
|------------|-----|---|-------|
| a | i | Photons are the massless particles of light✓ Whose energy is given by $E = hf$ where f is the frequency of light and h is Planck's constant✓ | 2 |
| a | ii |  <p>Horizontal , straight line ✓ Beginning at the critical frequency ✓</p> | 2 |
| a | iii | The photocurrent is the rate of emission of electrons from the photosurface times the electron's charge, ✓ And this is independent of photon frequency or the electron speed✓ | 2 |
| b | i | One of Emission without delay✓ Electron energy increases with photon frequency Existence of a critical frequency below which no electrons are emitted | 1 |
| b | ii | Using the first feature: With very weak electromagnetic waves incident on the surface an electron would have to accumulate energy slowly and so would take a long time to leave the metal✓ In the photon model of light an electron absorbs all the energy of the photon at once and so there is no delay✓ Using the second feature: The energy of an electromagnetic waves does not depend on frequency✓ But the energy of a photon increases with frequency✓ Using the third feature: The energy of an electromagnetic waves does not depend on frequency✓ But the energy of a photon does and if the frequency is low the supplied energy cannot overcome the work function so no electrons are emitted✓ | 2 |
| c | i | Extending the graph to the vertical intercept gives -3.4 V ✓ So, the work function is 3.4 eV ✓ | 2 |
| c | ii | From $E = hf - \phi$ and $E = eV$ we have that $V = \frac{h}{e}f - \frac{\phi}{e}$ and so the gradient of the graph is the Planck constant divided by e ✓ | 3 |

| | | | |
|----------|------------|---|----------|
| | | Gradient is $\frac{8.0-0}{3.0 \times 10^{15} - 0.90 \times 10^{15}} = 3.8 \times 10^{-15} \text{ VHz}^{-1} \checkmark$ $h = 1.6 \times 10^{-19} \times 3.8 \times 10^{-15} = 6.1 \times 10^{-34} \text{ J s} \checkmark$ | |
| c | iii | The critical frequency is $9.0 \times 10^{14} \text{ Hz}$ (intercept on f axis) and so the corresponding wavelength is $\frac{3.0 \times 10^8}{9.0 \times 10^{14}} = 3.3 \times 10^{-7} \text{ m} \checkmark$ | 1 |
| d | | The energy of the emitted electrons does not depend on intensity \checkmark So, the graph will not change \checkmark | 2 |

4.

A beam of light of diameter 1.2 cm is incident on a metal surface. The wavelength of light is 640 nm and the intensity of the incident light is $2.4 \times 10^{-2} \text{ W m}^{-2}$.



- (a) Calculate the energy of one photon of this light. [1]
- (b) Determine the number of photons per second incident on the surface. [3]
- (c) The photons are all absorbed by the surface.
 - (i) Calculate the momentum of one of the photons. [1]
 - (ii) Hence determine the pressure that the light exerts on the surface. [2]
- (d) Calculate the current leaving the surface if each of the incident photons ejects one electron. [2]
- (e) (i) Estimate the time needed for an energy of 1 eV to be absorbed by a circular area of radius 10^{-10} m . [2]
 - (ii) Comment on the significance of the result in (e) (i). [2]

| Question 4 | | Answers | Marks |
|------------|----|---|-------|
| a | | $E = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3.0 \times 10^8}{640 \times 10^{-9}} = 3.11 \times 10^{-19} \approx 3.1 \times 10^{-19} \text{ J} \checkmark$ | 1 |
| b | | <p>The area upon which the light is incident is $\pi r^2 = \pi \left(\frac{0.012}{2} \right)^2 = 1.13 \times 10^{-4} \text{ m}^2 \checkmark$</p> <p>The energy received by this area in 1 s is $2.4 \times 10^{-2} \times 1.13 \times 10^{-4} = 2.71 \times 10^{-6} \text{ J} \checkmark$</p> <p>Hence the number of photons incident is $N = \frac{2.71 \times 10^{-6}}{3.11 \times 10^{-19}} = 8.73 \times 10^{12} \approx 8.7 \times 10^{12} \checkmark$</p> | 3 |
| c | i | $p = \frac{h}{\lambda} = \frac{6.63 \times 10^{-34}}{640 \times 10^{-9}} = 1.036 \times 10^{-27} \approx 1.0 \times 10^{-27} \text{ N s} \checkmark$ | 1 |
| c | ii | <p>In 1 s the change in the incident photons' momentum is</p> <p>$8.73 \times 10^{12} \times 1.036 \times 10^{-27} = 9.04 \times 10^{-15} \approx 9.0 \times 10^{-15} \text{ N}$ which is the force the light exerts on the surface \checkmark</p> <p>Hence the pressure is $\frac{9.04 \times 10^{-15}}{1.13 \times 10^{-4}} = 8.0 \times 10^{-11} \text{ Pa} \checkmark$</p> | 2 |
| d | | $8.73 \times 10^{12} \times 1.6 \times 10^{-19} = 1.4 \mu\text{A} \checkmark$ | 1 |
| e | i | <p>$2.4 \times 10^{-2} \times \pi \times (10^{-10})^2 \times t = 1.0 \times 1.6 \times 10^{-19} \checkmark$</p> <p>$t = \frac{1.0 \times 1.6 \times 10^{-19}}{2.4 \times 10^{-2} \times \pi \times (10^{-10})^2} = 212 \text{ s} \approx 3.5 \text{ min} \checkmark$</p> | 2 |
| e | ii | <p>In the wave theory of light, it would take considerable time for an electron to accumulate enough energy to escape \checkmark</p> <p>Contrary to the observed lack of time delay in the emission of electrons \checkmark</p> | 2 |

5.

In a hydrogen atom an electron of mass m orbits the proton with speed v in a circular orbit of radius r . The Bohr condition $mvr = n\frac{h}{2\pi}$ implies that $r = a_0 n^2$ where $a_0 = 0.53 \times 10^{-10}$ m is known as the Bohr radius.

(a) Deduce that $v = \frac{h}{2\pi m a_0} \times \frac{1}{n}$. [2]

(b) Determine the ratio of the electron speed in the orbit $n = 3$ to that in $n = 2$. [1]

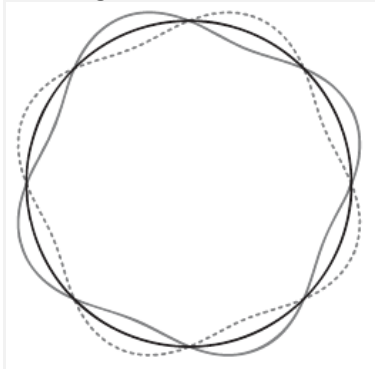
(c)

(i) Show that the frequency of revolution in the n^{th} state is given by $f = \frac{h}{4\pi^2 m a_0^2} \frac{1}{n^3}$. [2]

(ii) Calculate this frequency for an electron in the state $n = 2$ of hydrogen. [2]

(d) Demonstrate that the Bohr condition is equivalent to $2\pi r = n\lambda$ where λ is the de Broglie wavelength of the electron. [2]

The diagram shows an electron wave in hydrogen.



(e)

(i) State what is meant by an *electron wave*. [1]

(ii) Determine the radius of the circular orbit of this electron. [3]

(iii) Predict the energy that must be supplied for this electron to become free. [1]

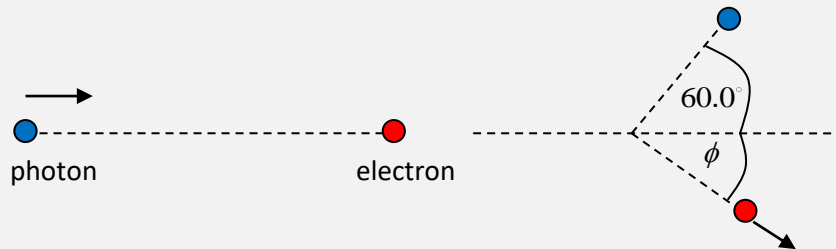
| Question 5 | | Answers | Marks |
|------------|-----|--|-------|
| a | i | <p>Use of $mvr = n \frac{h}{2\pi}$ ✓</p> <p>$mva_0 n^2 = n \frac{h}{2\pi}$ ✓</p> <p>To give answer $v = \frac{h}{2\pi m a_0} \frac{1}{n}$.</p> | 2 |
| b | | $\frac{v_3}{v_2} = \frac{\frac{h}{2\pi m a_0} \times \frac{1}{3}}{\frac{h}{2\pi m a_0} \times \frac{1}{2}} = \frac{2}{3}$ ✓ | 1 |
| c | i | <p>$f = \frac{1}{T} = \frac{v}{2\pi r}$ ✓</p> <p>$f = \frac{\frac{h}{2\pi m a_0} \frac{1}{n}}{2\pi a_0 n^2}$ ✓</p> <p>To give answer $f = \frac{h}{4\pi^2 m a_0^2} \times \frac{1}{n^3}$.</p> | 2 |
| c | ii | <p>$f = \frac{6.63 \times 10^{-34}}{4\pi^2 \times 9.1 \times 10^{-31} \times (0.53 \times 10^{-10})^2} \times \frac{1}{2^3}$ ✓</p> <p>$f = 8.2 \times 10^{14}$ Hz ✓</p> | 2 |
| d | | <p>$mvr = n \frac{h}{2\pi}$ and $\lambda = \frac{h}{p}$</p> <p>$pr = n \frac{h}{2\pi}$ ✓</p> <p>$2\pi r = n \frac{h}{p}$ ✓</p> <p>Leading to answer.</p> | 2 |
| e | i | An electron wave is a wave whose amplitude is related to the probability of finding the electron in a region of space at some time ✓ | 1 |
| e | ii | <p>This corresponds to $n = 4$ ✓</p> <p>$r_4 = n^2 a_0 = 4^2 \times 0.53 \times 10^{-10} = 8.5 \times 10^{-10}$ m ✓</p> | 2 |
| e | iii | $E_4 = -\frac{13.6}{4^2} = -0.85$ eV is the minimum energy that must be supplied ✓ | 1 |

6.

(a) Suggest how the photoelectric effect and the Compton effect provide evidence for the particle nature of light. [2]

(b) A photon of wavelength 2.80×10^{-12} m scatters off an electron at rest at an angle of 60.0° to the original path of the photon.

The electron moves off at an angle ϕ with momentum 2.10×10^{-22} N s.



Calculate

(i) the wavelength of the scattered photon. [2]

(ii) the energy transferred to the electron. [2]

(c) Determine, to two significant figures, ϕ . [3]

| Question 6 | | Answers | Marks |
|------------|----|--|-------|
| a | | <p>The photoelectric effect provides evidence for light as a particle of energy hf ✓</p> <p>The Compton provides evidence for light as a particle of energy hf and momentum $\frac{h}{\lambda}$ ✓</p> | 2 |
| b | i | $\lambda' - \lambda = \frac{h}{mc}(1 - \cos\theta) \text{ so}$ $\lambda' - 2.80 \times 10^{-12} = \frac{6.63 \times 10^{-34}}{9.11 \times 10^{-31} \times 3.0 \times 10^8} \left(1 - \frac{1}{2}\right) = 1.213 \times 10^{-12} \text{ m} \checkmark$ $\lambda' = 4.013 \times 10^{-12} \approx 4.01 \times 10^{-12} \text{ m} \checkmark$ | 2 |
| b | ii | $\Delta E = \frac{hc}{\lambda} - \frac{hc}{\lambda'} = \frac{1.24 \times 10^{-6}}{2.80 \times 10^{-12}} - \frac{1.24 \times 10^{-6}}{4.01 \times 10^{-12}} = 133.63 \approx 134 \text{ keV} \checkmark$ | 2 |
| c | | <p>Alternative 1</p> <p>Momentum conservation gives in y direction:</p> <p>Momentum of scattered photon is $\frac{h}{\lambda'} = \frac{6.63 \times 10^{-34}}{4.013 \times 10^{-12}} = 1.652 \times 10^{-22} \text{ m} \checkmark$</p> $1.652 \times 10^{-22} \times \sin 60^\circ = 2.09585 \times 10^{-22} \times \sin \phi \checkmark$ $\sin \phi = \frac{1.652 \times 10^{-22} \times \sin 60^\circ}{2.10 \times 10^{-22}} = 0.6813 \Rightarrow \phi = 43^\circ \checkmark$ <p>Alternative 2</p> <p>Momentum conservation gives in x direction:</p> <p>Momenta of photons: $\frac{h}{\lambda'} = \frac{6.63 \times 10^{-34}}{4.013 \times 10^{-12}} = 1.652 \times 10^{-22} \text{ m}$ and</p> $\frac{h}{\lambda} = \frac{6.63 \times 10^{-34}}{2.80 \times 10^{-12}} = 2.368 \times 10^{-22} \text{ m} \checkmark$ $2.368 \times 10^{-22} = 1.652 \times 10^{-22} \times \cos 60^\circ + 2.10 \times 10^{-22} \times \cos \phi \checkmark$ $\cos \phi = \frac{2.368 \times 10^{-22} - 1.652 \times 10^{-22} \times \cos 60^\circ}{2.10 \times 10^{-22}} = 0.7343 \Rightarrow \phi = 43^\circ \checkmark$ | 3 |

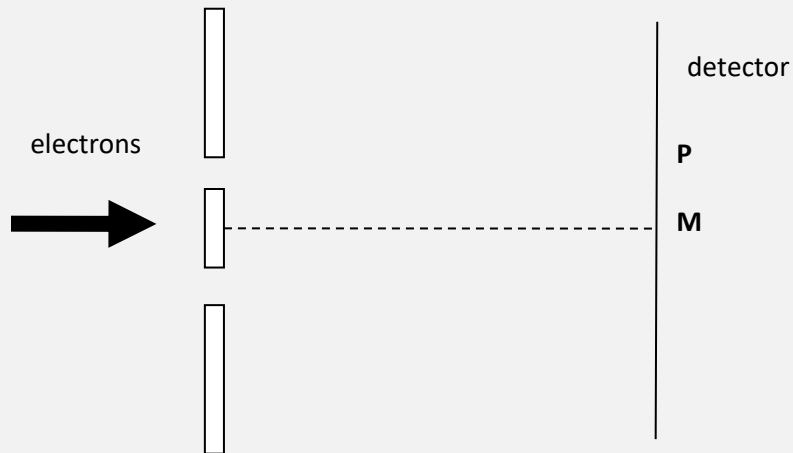
7.

- (a) Outline what is meant by the *de Broglie hypothesis*. [2]
- (b)
- (i) Show that the de Broglie wavelength of an electron that has been accelerated from rest by a potential difference V is given by $\lambda = \frac{h}{\sqrt{2meV}}$. [2]
- (ii) Calculate the de Broglie wavelength of an electron that has been accelerated from rest by a potential difference of 120 V. [1]
- (c) Describe an experiment in which the de Broglie hypothesis is tested. [3]
- (d) A bullet of mass 0.080 kg leaves a gun with speed 420 m s^{-1} . The gun is in perfect condition and has been fired by an expert marksman. The bullet must pass through a slit of width 5.0 cm on its way to its target. A student says that the bullet will miss its target because of de Broglie's hypothesis. By suitable calculations determine whether the student is correct. [3]

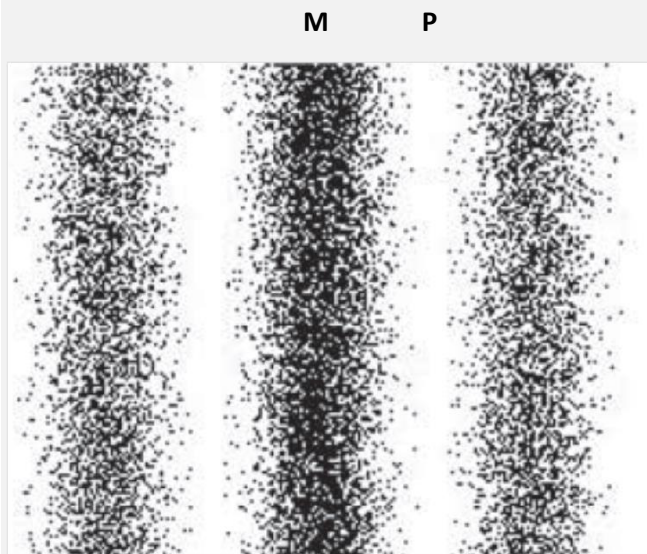
| Question 7 | | Answers | Marks |
|------------|-----------|---|----------|
| a | | All moving particles have wavelike properties✓ With a wavelength given by $\lambda = \frac{h}{p}$ ✓ | 2 |
| b | i | $E_k = eV$ ✓ $E_k = \frac{p^2}{2m} \Rightarrow p = \sqrt{2mE_k} = \sqrt{2meV}$ ✓ Substitute in $\lambda = \frac{h}{p}$ to get answer. | 2 |
| b | ii | $\lambda = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 120 \times 1.6 \times 10^{-19}}} = 1.1 \times 10^{-10} \text{ m}$ ✓ | 1 |
| c | | See textbook. | 3 |
| d | | The de Broglie wavelength of the bullet is $\lambda = \frac{6.63 \times 10^{-34}}{0.080 \times 420} \approx 2 \times 10^{-35} \text{ m}$ ✓ This is way too small to show diffraction effects through a 5 cm hole✓ The bullet will hit the target✓ | 3 |

8.

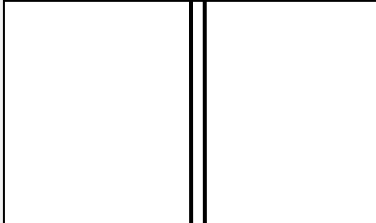
Electrons are directed at two slits and are observed by a detector some distance away.



The detector records the number of electrons incident on it. The following pattern was observed, where each dot represents a detected electron.



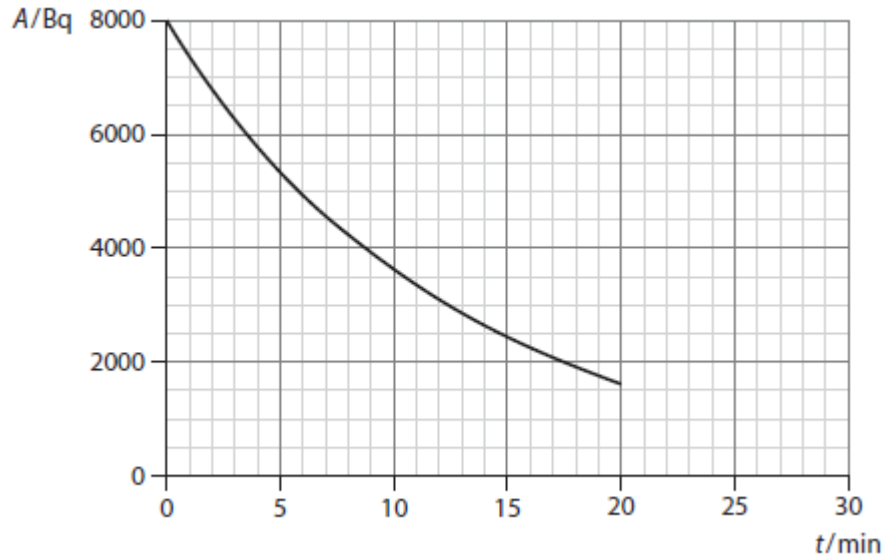
- (a) Explain how this pattern is evidence in support of the wave nature of the electron. [2]
- (b) Draw a diagram to show the pattern that would be expected if the electrons behaved as particles. [1]
- (c) The path difference at P is $3.0 \times 10^{-11} \text{ m}$. The kinetic energy of the electrons is 420 eV. Explain how these data support the wave nature of the electron. [4]

| Question 8 | | Answers | Marks |
|------------|---|--|-------|
| a | | The diagram is a typical interference pattern✓ Interference is a fundamental property of waves✓ | 2 |
| b | |  Diagram as shown✓ | 1 |
| c | i | $E_k = \frac{p^2}{2m} \Rightarrow p = \sqrt{2mE_k} \quad \checkmark$ $p = \sqrt{2 \times 9.1 \times 10^{-31} \times 420 \times 1.6 \times 10^{-19}} = 1.106 \times 10^{-23} \text{ N s } \checkmark$ $\lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34}}{1.106 \times 10^{-23}} = 5.995 \times 10^{-11} \approx 6.0 \times 10^{-11} \text{ m } \checkmark$ <p>P is the first minimum of the interference pattern and so path difference is</p> $(0 + \frac{1}{2})\lambda = 3.0 \times 10^{-11} \text{ m as stated } \checkmark$ | 4 |

9.

(a) Radioactive decay is random and spontaneous. State what you understand by this statement. [3]

(b) The graph shows how activity of a sample containing a radioactive isotope of thorium $^{225}_{90}\text{Th}$ varies with time.



- (i) State what is meant by an *isotope*. [1]
- (ii) Determine the half-life of thorium. [2]
- (iii) State one assumption made in obtaining the answer to (ii). [1]
- (iv) Draw a graph to show the variation of the activity with time up to 30 minutes. [1]

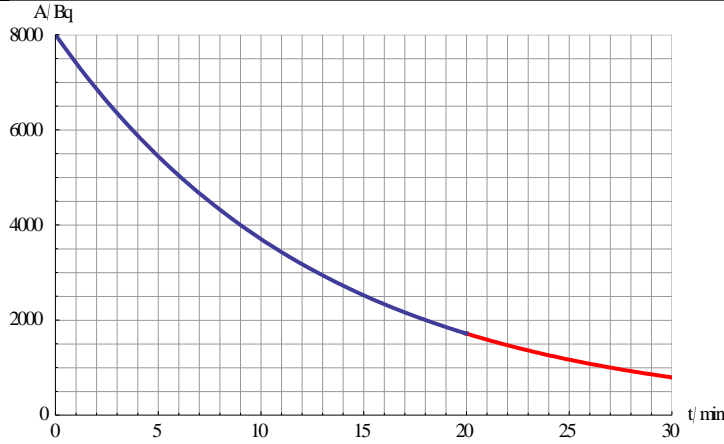
(c) (i) Thorium undergoes alpha decay. Complete the reaction equation:



Atomic masses: thorium 225.02395 u, radium 221.013917 u, helium 4.0026603 u.

- (ii) Calculate, in kg, the mass that has been converted into energy in this reaction. [2]
- (iii) Calculate, in MeV, the energy released. [1]

(d) The nuclei of thorium are at rest when they decay. Determine the fraction of the energy released that is carried by the alpha particle. [3]

| Question 9 | | Answers | Marks |
|------------|-----|---|-------|
| a | | Random: We cannot predict which nucleus will decay✓ Or when it will decay✓ Spontaneous: The decay will happen independently of any action from the outside✓ | 3 |
| b | i | A nucleus of the same chemical element i.e. with the same number of protons but differing in the number of neutrons✓ | 1 |
| b | ii | Finding the point with half the initial activity✓ 9.0 min✓ | 2 |
| b | iii | Ignored a possible background activity/Assumed daughter nucleus is stable so it does not contribute to activity✓ | 1 |
| b | iv |  Shape shown✓ | 1 |
| c | i | ${}_{90}^{225}\text{Th} \rightarrow {}_{88}^{221}\text{Ra} + {}_2^4\alpha$ Alpha numbers✓ Radium numbers✓ | 2 |
| c | ii | Mass difference: $225.02395 - 221.013917 - 4.0026603 = 7.3727 \times 10^{-3} \text{ u} \checkmark$ $7.3727 \times 10^{-3} \times 1.661 \times 10^{-27} = 1.22 \times 10^{-29} \text{ kg} \checkmark$ | 2 |
| c | iii | $7.3727 \times 10^{-3} \times 931.5 = 6.87 \text{ MeV} \checkmark$ | 1 |
| d | | The alpha and radium have equal and opposite momenta✓ $\text{Ratio of energies is } \frac{K_\alpha}{K_R} = \frac{\frac{p^2}{2m_\alpha}}{\frac{p^2}{2m_R}} = \frac{m_R}{m_\alpha} \approx \frac{221}{4} \approx 55 \checkmark$ Alpha particle energy fraction is then $\frac{55}{56} \checkmark$ | 3 |

10.

(a) State what is meant by the *binding energy* of a nucleus. [1]

(b) Calculate the binding energy per nucleon of the stable nucleus $^{14}_7\text{N}$. The nuclear mass of $^{14}_7\text{N}$ is 13.9992 u. [3]

(c) $^{17}_7\text{N}$ is an unstable isotope of $^{14}_7\text{N}$.

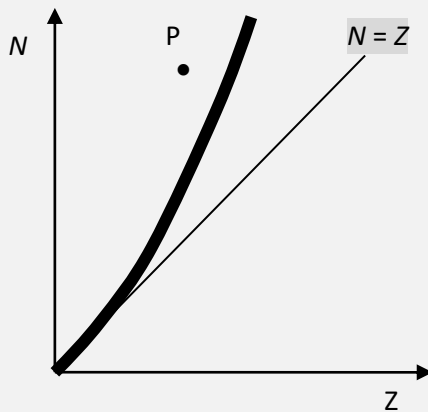
(i) State what is meant by the term *isotopes*. [1]

(ii) Suggest how the binding energy per nucleon for $^{17}_7\text{N}$ compares with that of $^{14}_7\text{N}$. [2]

(iii) $^{17}_7\text{N}$ decays to the nuclide $^{13}_6\text{C}$ by a sequence of two decays. State the types of decay taking place. [2]

(d) Suggest why most nuclei with $A > 60$ have roughly the same binding energy per nucleon. [3]

(e) The dark band in the graph of neutron number N against proton number Z , shows the position of stable nuclei. The straight line is the equation $N = Z$.



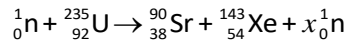
(i) Suggest why large stable nuclei have more neutrons than protons. [3]

(ii) An unstable nucleus occupies position P in the graph. Suggest the likely decay for this nucleus. [2]

| Question 10 | | Answers | Marks |
|-------------|-----|--|-------|
| a | | The minimum energy that must be supplied to a nucleus so that the nucleons are all separated✓ OR The energy released when a nucleus is assembled from its constituent nucleons ✓ | 1 |
| b | | Mass defect: $7m_p + 7m_n - M = 7 \times 1.007276 + 7 \times 1.008665 - 13.9992 = 0.112387 \text{ u}$ ✓ Binding energy: $0.112387 \times 931.5 = 104.688 \text{ MeV}$ ✓ BE per nucleon: $104.688/14 = 7.478 \text{ MeV}$ ✓ | 3 |
| c | i | Nuclei of the same chemical element i.e. with the same number of protons but differing in the number of neutrons✓ | 1 |
| c | ii | Binding energy per nucleons is a measure of how tightly bound a nucleus is✓ So, it is larger for the stable nucleus ${}^{14}_7\text{N}$ ✓ (In fact the binding energy per nucleon for ${}^{17}_7\text{N}$ 7.286 MeV.) | 2 |
| c | iii | One alpha decay and one beta minus decay must be involved. Two possibilities are: ${}^{17}_7\text{N} \rightarrow {}^{17}_8\text{O} + {}^0_{-1}\text{e} + \bar{\nu}$ ✓ ${}^{17}_8\text{O} \rightarrow {}^{13}_6\text{C} + {}^4_2\alpha$ ✓ Or ${}^{17}_7\text{N} \rightarrow {}^{13}_5\text{B} + {}^4_2\alpha$ ✓ ${}^{13}_5\text{B} \rightarrow {}^{13}_6\text{C} + {}^0_{-1}\text{e} + \bar{\nu}$ ✓ | 2 |
| d | | In a large enough nucleus, any given nucleon is surrounded by the same number of immediate neighbouring nucleons✓ Only these nucleons attract the given nucleon because the strong force has a short range✓ Since binding energy per nucleons is roughly the energy to free a nucleon this quantity is approximately the same for all large nuclei ✓ | 3 |
| e | i | A nucleus gets bigger as more protons and neutrons are added✓ Protons repel each other with the long range electric force so protons tend to destabilize a nucleus✓ Adding extra neutrons increases the distance between protons thus reducing the repelling forces and contributes to binding through the nuclear force✓ | 3 |
| e | ii | Beta minus decay✓ Because this would lower the position of P in the graph by 1 unit and move it to the right by 1 unit, i.e. closer to the band of stability✓ | 2 |

11.

A fission reaction is given by the equation:



- (a) (i) Suggest whether this is an example of spontaneous or induced fission. [1]
(ii) Determine the number x of neutrons produced. [1]

- (b) The following data are available for binding energy per nucleon:

U: 7.59 MeV; Sr: 8.70 MeV; Xe: 8.20 MeV

Show that the energy released in this reaction is about 170 MeV. [2]

A nuclear fission reactor produces energy through the reaction above. The reactor produces 580 MW of electrical power with an overall efficiency of 38%.

- (c) Estimate [4]
(i) the mass of uranium-235 that must undergo fission in one year. [4]
(ii) the mass of radioactive waste collected in one year. [1]
- (d) Suggest why special precautions must be taken for the storage of nuclear waste. [3]
- (e) Describe the function of the following elements of a nuclear fission reactor:
- (i) moderator [2]
(ii) control rods [2]
(iii) heat exchanger [2]

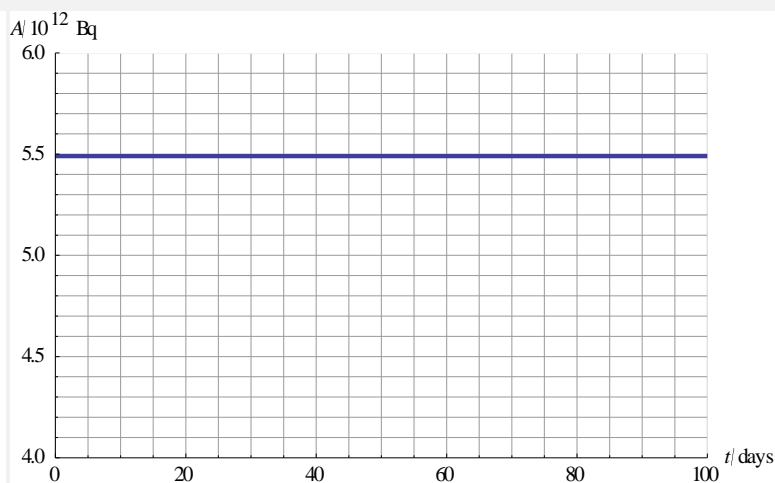
| Question 11 | | Answers | Marks |
|-------------|-----|--|-------|
| a | i | Induced since a neutron is involved✓ | 1 |
| a | ii | 3 ✓ | 1 |
| b | | Q = difference in binding energy (right minus left)✓ $Q = 90 \times 8.70 + 143 \times 8.20 - 235 \times 7.59 = 171.9 \text{ MeV}$ ✓ | 2 |
| c | i | Power produced by fission is $\frac{580}{0.38} = 1526 \text{ MW}$ ✓ Energy produced in a year $1526 \times 10^6 \times 365 \times 24 \times 60 \times 60 = 4.812 \times 10^{16} \text{ J}$ ✓ Number of reactions needed: $\frac{4.812 \times 10^{16}}{171.9 \times 10^6 \times 1.6 \times 10^{-19}} = 1.7496 \times 10^{27}$ ✓ Mass in a year: $1.7496 \times 10^{27} \times 235 \times 1.661 \times 10^{-27} \approx 683 \text{ kg}$ ✓ | 4 |
| c | ii | Sr: $1.7496 \times 10^{27} \times 90 \times 1.661 \times 10^{-27} \approx 262 \text{ kg}$ Xe: $1.7496 \times 10^{27} \times 143 \times 1.661 \times 10^{-27} \approx 416 \text{ kg}$ For a total of 678 kg✓ | 1 |
| d | | The waste products are highly radioactive, producing penetrating radiation decades after they have been removed from the reactor ✓ Leakage of this material into groundwater, lakes and rivers may enter the food chain contaminating many people and animals ✓ Because the energy produced generates heat so storage must ensure no leaks and cooling to avoid fires ✓ | 3 |
| e | i | Slows down neutrons emitted in the fission reactions✓ So that these neutrons can be used to induce fission of other nuclei✓ | 2 |
| e | ii | Absorb neutrons when too many reactions are taking place✓ This controlling the rate of reactions✓ | 2 |
| e | iii | The collision of neutrons with moderator atoms increases the temperature of the moderator✓ The generated thermal energy is removed from the moderator by the heat exchanger which could be water circulating inside the moderator✓ | 2 |

12.

The half-life of radium $^{226}_{88}\text{Ra}$ is 1600 years. Radium decays into radon (Rn). Radon is unstable with a half-life of 3.8 days.

(a) Show that the activity of 1.5 mg of radium is about 5.5×10^7 Bq. [3]

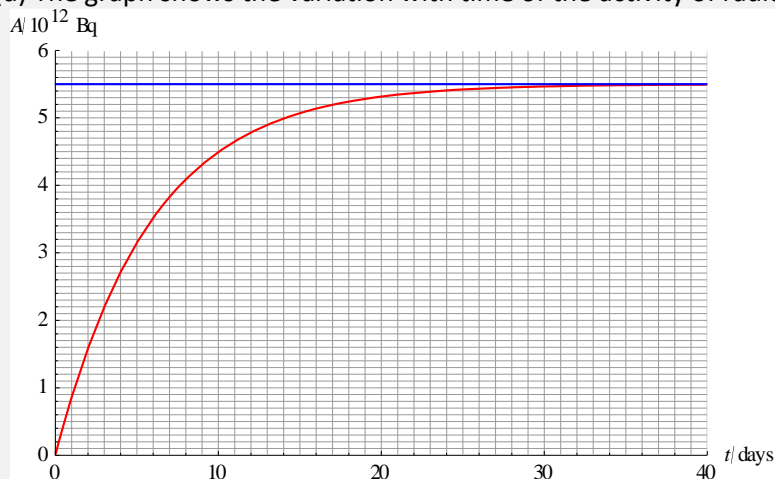
(b) The graph shows the activity of radium as a function of time in days.



Explain why the graph appears to be a horizontal line. [2]

(c) The decay of radium into radon is an alpha decay. Write down the equation of the decay. [1]

(d) The graph shows the variation with time of the activity of radium (blue) and radon (red).



(i) Explain what can be deduced from the fact that the activity of radon eventually becomes constant. [2]

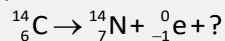
(ii) Estimate the mass of radon present after 30 days. [3]

(e) The half-life of an isotope is 1.0 minute. Use the concept of the decay constant as a probability per unit time, to show that the probability of decay within one minute is 0.50. [3]

| Question 12 | | Answers | Notes | Marks |
|-------------|----|--|-------|-------|
| a | | $\lambda = \frac{\ln 2}{1600 \times 365 \times 24 \times 60 \times 60} = 1.374 \times 10^{-11} \text{ s}^{-1} \checkmark$ <p>Number of nuclei initially: $\frac{1.5 \times 10^{-3}}{226} \times 6.02 \times 10^{23} = 3.9956 \times 10^{18} \checkmark$</p> <p>Initial activity: $\lambda N_0 = 1.374 \times 10^{-11} \times 3.9956 \times 10^{18} = 5.49 \times 10^7 \text{ Bq} \checkmark$</p> | | 3 |
| b | | <p>The half-life of radium is much longer than the interval of 100 days shown in the graph✓</p> <p>During 100 days the activity would decrease by an extremely small amount which cannot be shown on the graph✓</p> | | 2 |
| c | | ${}_{88}^{226}\text{Ra} \rightarrow {}_{86}^{222}\text{Rn} + {}_2^4\alpha \checkmark$ | | 1 |
| d | i | <p>The number of radon nuclei decreases because radon is radioactive and decays away but also increases because radon is produced in the decay of radium✓</p> <p>Since the activity becomes constant eventually it means that the rate at which radon is produced is equal to the rate at which it decays✓</p> | | 2 |
| d | ii | <p>Radon decay constant is $\lambda = \frac{\ln 2}{3.8 \times 24 \times 60 \times 60} = 2.11 \times 10^{-6} \text{ s}^{-1} \checkmark$</p> <p>After 30 days: $2.11 \times 10^{-6} \times N = 5.49 \times 10^7 \text{ Bq} \Rightarrow N = 2.60 \times 10^{13}$ or $\frac{2.60 \times 10^{13}}{6.02 \times 10^{23}} = 4.32 \times 10^{-11} \text{ mol} \checkmark$</p> <p>Hence a mass of $4.32 \times 10^{-11} \times 222 = 9.6 \times 10^{-9} \text{ g} \checkmark$</p> | | 3 |
| e | | <p>Express λ in units where it is numerically much smaller than 1:</p> $\lambda = \frac{\ln 2}{1.0} \text{ min}^{-1} = 0.6931 \text{ min}^{-1} = 1.155 \times 10^{-2} \text{ s}^{-1} = 1.155 \times 10^{-5} \text{ ms}^{-1} \checkmark$ <p>$(1 - 1.155 \times 10^{-5})$ is the probability of no decay in 1 ms and $(1 - 1.155 \times 10^{-5})^{1000 \times 60} = 0.499998$ is the probability of no decay in 1 min✓</p> <p>Hence $1 - 0.499998 = 0.500002 \approx 0.50$ is the probability of decay within 1 min✓</p> | | 3 |

13.

Carbon-14 is unstable and decays to nitrogen by beta minus emission according to the reaction equation:



(a) State, for the missing particle in the reaction equation:

(i) its name

[1]

(ii) two of its properties.

[2]

(b) Outline the evidence that made the presence of this particle in beta decay necessary.

[2]

In a living tree, the ratio of carbon-14 to carbon-12 atoms is constant at 1.3×10^{-12} . The half-life of ${}^{14}_{6}\text{C}$ is 5730 years.

(c) Suggest why this ratio will decrease after the tree dies.

[2]

A 15 g piece of charcoal is found at an archaeological site.

(d) Calculate the number of atoms of carbon-12 in the piece of charcoal.

[2]

(e) (i) The piece of charcoal has an activity of 1.40 Bq. Deduce that the ratio of carbon-14 to carbon-12 atoms in the charcoal is 4.85×10^{-13} .

[3]

(ii) Deduce the age of the charcoal.

[3]

| Question 13 | | Answers | Marks |
|-------------|----|---|-------|
| a | i | Anti-neutrino✓ | 1 |
| a | ii | Two of: Electrically neutral✓ Almost massless✓ Very weakly interacting✓ Spin $\frac{1}{2}$ ✓ | 2 |
| b | | If the neutrino was not there the electron would be carrying most of the energy Q produced in the decay✓ Experiments show that the electron carries anything from 0 to Q so a third particle must be present to take the rest of the energy✓ | 2 |
| c | | Biological processes keep the ratio constant when the tree is alive✓ After the tree dies the radioactive C-14 decays away so the ratio is reduced✓ | 2 |
| d | | $\frac{15}{12} = 1.25 \text{ mol}$ ✓ So $N = 1.25 \times 6.02 \times 10^{23} = 7.525 \times 10^{23}$ ✓ [It might be objected that the calculation above assumes that all the carbon in the 15 g is C-12. The maximum possible number of C-14 originally present is $1.3 \times 10^{-12} \times N$ where N is the number of C-12 atoms. The mass of C-12 atoms is 12.0000000 u and that of C-14 14.00324199 u. Then $(N \times 12 + 1.3 \times 10^{-12} \times N \times 14.00324199) \times 1.661 \times 10^{-27} = 15 \times 10^{-3}$ and so $N = 7.526 \times 10^{23}$ so the error is negligible .] | 2 |
| e | i | $\lambda = \frac{\ln 2}{5730 \times 365 \times 24 \times 60 \times 60} = 3.836 \times 10^{-12} \text{ s}^{-1}$ ✓ $\lambda N = 3.836 \times 10^{-12} \times N = 1.40 \text{ Bq} \Rightarrow N = 3.650 \times 10^{11}$ ✓ Ratio of C-14 to C-12 is $\frac{3.650 \times 10^{11}}{7.52 \times 10^{23}} = 4.8537 \times 10^{-13} \approx 4.85 \times 10^{-13}$ ✓ | 3 |
| e | ii | Activity is proportional to the number of nuclei present✓ $4.85 \times 10^{-13} = 1.3 \times 10^{-12} \times e^{-1.21 \times 10^{-4} t}$ (decay constant in inverse years)✓ Solving, $t = 8150 \text{ years}$ ✓ | 3 |

14.

(a) Outline the evidence in support of *nuclear energy levels*.

[2]

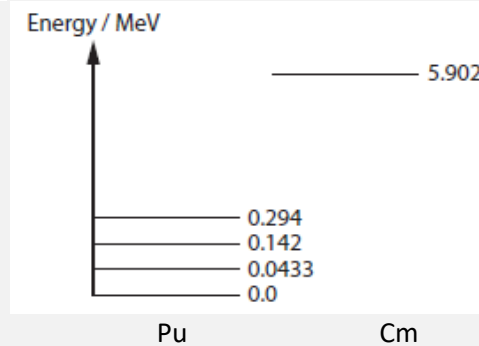
(b) The diagram shows nuclear energy levels for $^{244}_{96}\text{Cm}$ and $^{240}_{94}\text{Pu}$.

(i) On a copy of the diagram, indicate the alpha decay of $^{244}_{96}\text{Cm}$ into $^{240}_{94}\text{Pu}$ that is followed by the emission of a photon of energy 0.043 MeV.

[1]

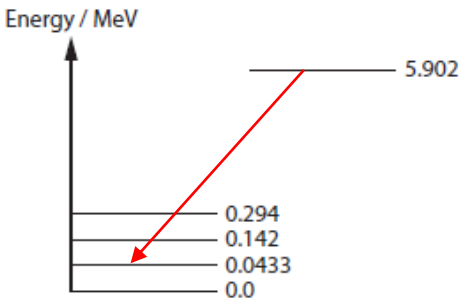
(ii) Deduce the energy of the emitted alpha particle.

[1]



(iii) Calculate the wavelength of the emitted photon.

[2]

| Question 14 | | Answers | Marks |
|-------------|------------|---|----------|
| a | | The discrete energy of alpha particles in radioactive decay ✓ The discrete energy of gamma rays in radioactive decay ✓ | 2 |
| b | i |  | 1 |
| b | ii | $5.902 - 0.0433 = 5.8587 \text{ MeV} \checkmark$ | 1 |
| b | iii | $E = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{E} \checkmark$ $\lambda = \frac{1.24 \times 10^{-6}}{0.0433 \times 10^6} = 2.86 \times 10^{-11} \text{ m} \checkmark$ | 2 |

15.

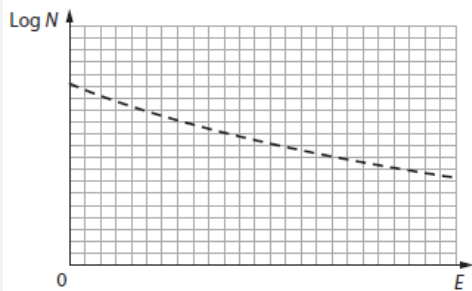
(a) Show that the density of all nuclei is about $2 \times 10^{17} \text{ kg m}^{-3}$. [3]

(b) (i) In Rutherford scattering, alpha particles of energy 5.2 MeV are directed head-on at a lead nucleus ($^{202}_{88}\text{Pb}$). Estimate the distance of closest approach between the alpha particles and the centre of the lead nucleus. [3]

(ii) Determine the distance between the point of closest approach and the nuclear surface. [2]

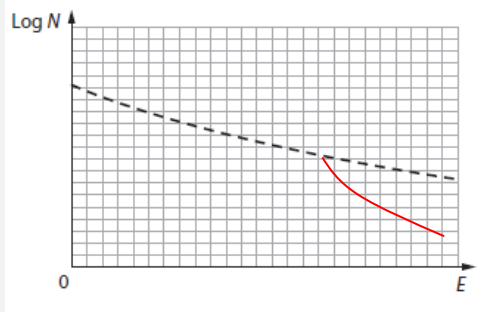
(iii) Calculate the acceleration of the alpha particle when at the point of closest approach. The mass of an alpha particle is $6.64 \times 10^{-27} \text{ kg}$. [2]

(c) The graph shows how the number of alpha particles that are observed at a fixed scattering angle depends on alpha particle energy according to Rutherford's calculations.



(i) State one assumption the Rutherford calculations are based on. [1]

(ii) On a copy of the diagram above, indicate deviations from the Rutherford scattering. Explain your answer. [2]

| Question 15 | | Answers | Marks |
|-------------|-----|---|-------|
| a | | $\rho = \frac{M}{V} = \frac{M}{\frac{4\pi}{3}R^3} = \frac{3M}{4\pi R^3} \checkmark$ $R = 1.2 \times 10^{-15} \times A^{1/3} \text{ m and } M \approx Au \checkmark$ $\rho = \frac{3 \times A \times 1.661 \times 10^{-27}}{4\pi \times (1.2 \times 10^{-15} \times A^{1/3})^3} = 2.3 \times 10^{17} \text{ kg m}^{-3} \checkmark$ | 3 |
| b | i | $E_k = \frac{kQq}{r} \Rightarrow r = \frac{kQq}{E_k} \checkmark$ $r = \frac{8.99 \times 10^9 \times 2 \times 88 \times (1.6 \times 10^{-19})^2}{5.2 \times 10^6 \times 1.6 \times 10^{-19}} \checkmark$ $r = 4.87 \times 10^{-14} \approx 4.9 \times 10^{-14} \text{ m} \checkmark$ | 3 |
| b | ii | $R = 1.2 \times 10^{-15} \times A^{1/3} \text{ m} = 1.2 \times 10^{-15} \times 202^{1/3} \text{ m} = 7.04 \times 10^{-15} \text{ m} \checkmark$ $4.87 \times 10^{-14} - 7.04 \times 10^{-15} = 4.17 \times 10^{-14} \approx 4.2 \times 10^{-14} \text{ m} \checkmark$ | 2 |
| b | iii | $F = \frac{8.99 \times 10^9 \times 2 \times 88 \times (1.6 \times 10^{-19})^2}{(4.87 \times 10^{-14})^2} = 17.08 \text{ N} \checkmark$ $a = \frac{F}{m} = \frac{17.08}{6.64 \times 10^{-27}} = 2.6 \times 10^{27} \text{ m s}^{-2} \checkmark$ | 2 |
| c | i | The only force acting on the alpha particle is the electric force✓ | 1 |
| | |  <p>Diagram✓</p> <p>As the energy increases the alpha particles approach closer to the nucleus and so the nuclear force acts on them, the nucleus absorbs some, thus reducing the number that is being scattered✓</p> | 2 |

16.

(a) Distinguish between nuclear *fusion* and nuclear *fission*. [2]

(b) By reference to the binding energy curve, explain why energy is released in nuclear fusion and nuclear fission. [2]

(c) (i) State the conditions for controlled nuclear fusion to take place. [3]

(ii) Explain one of the conditions you mentioned in (c) (i). [2]

(d) (i) Identify particles X and Y in the fusion reaction ${}^3_2\text{He} + {}^3_1\text{H} \rightarrow {}^4_2\text{He} + \text{X} + \text{Y}$. [2]

(ii) The following binding energies per nucleon are given:

He-3 = 2.572681 MeV; H-3 = 2.827266 MeV; He-4 = 7.073915 MeV

Calculate the energy released. [2]

| Question 16 | | Answers | Marks |
|-------------|-----------|--|----------|
| a | | In nuclear fusion two light nuclei join to form a larger nucleus releasing energy ✓ In nuclear fission a large nucleus splits into two smaller nuclei releasing energy ✓ | 2 |
| b | | Energy will be released when the products of a reaction have a higher binding energy per nucleon than the reactants ✓ This is the case for both fission and fusion ✓ | 2 |
| c | i | High temperature of the fuel ✓ High density ✓ High confinement time ✓ | 3 |
| c | ii | Temperature: Nuclei are positively charged so they repel each other through the electric force ✓ At high temperatures the nuclei would be moving fast enough to overcome the Coulomb repulsion and crash into each other ✓ Density: The number density of the nuclei (number of nuclei per unit volume) must be high ✓ so that there are lots of them around for fusion to occur ✓ Confinement time: The nuclei must be maintained at high temperature and density for a long time ✓ So that many fusion reactions to take place ✓ | 2 |
| d | i | Proton ✓ Neutron ✓ | 2 |
| d | ii | $Q = \text{difference in binding energy (right minus left)} \checkmark$ $Q = 4 \times 7.073915 - 3 \times 2.572681 - 3 \times 2.827266 = 12.1 \text{ MeV} \checkmark$ | 2 |

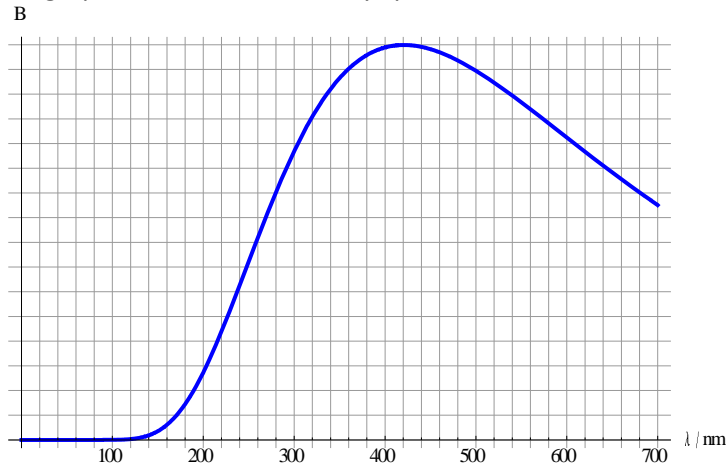
17.

- (a) Describe, with the help of a diagram, what is meant by the *parallax angle* of a star. [2]

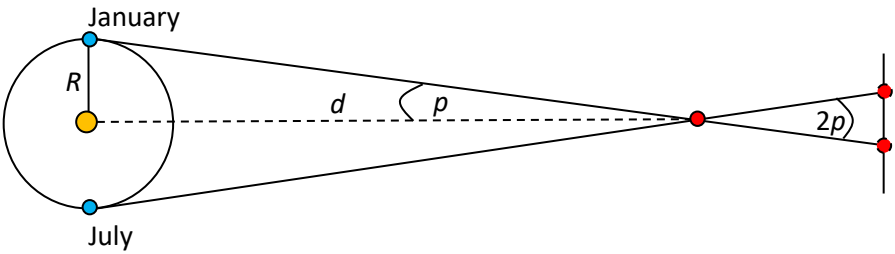
The parallax angle of a star S is 0.035 arcsec. The apparent brightness of S is $3.4 \times 10^{-10} \text{ W m}^{-2}$.

- (b) (i) Show that the distance to S is about $8.8 \times 10^{17} \text{ m}$. [2]
 (ii) Calculate the luminosity of S. [2]

- (c) The graph shows the black body spectrum for S.

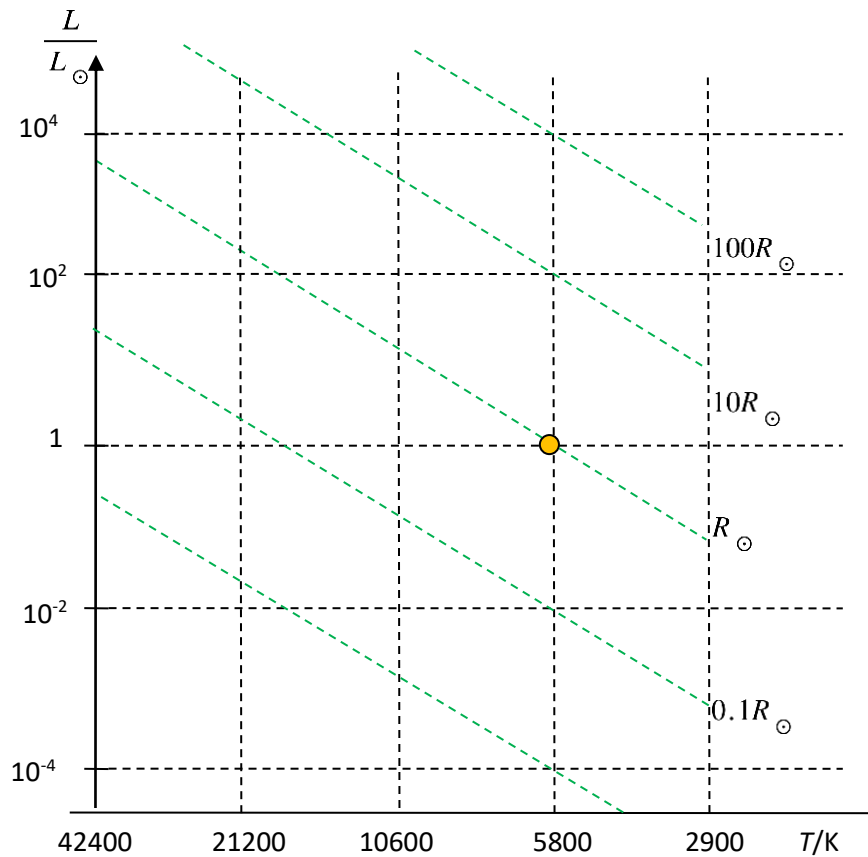


- (i) Estimate the surface temperature of S. [2]
 (ii) Calculate the radius of S. [3]

| Question 17 | | Answers | Marks |
|-------------|----|--|-------|
| a | |  <p>Diagram✓ Parallax is the angle at which a star subtends a distance equal to the radius of the Earth's orbit around the Sun✓</p> | 2 |
| b | i | $d = \frac{1}{p} = \frac{1}{0.035} = 28.57 \text{ pc} \checkmark$ $d = 28.57 \times 3.26 \times 9.46 \times 10^{15} = 8.81 \times 10^{17} \text{ m} \checkmark$ | 2 |
| b | ii | $b = \frac{L}{4\pi d^2} \Rightarrow L = 4\pi d^2 b \checkmark$ $L = 4\pi \times (8.81 \times 10^{17})^2 \times 3.4 \times 10^{-10} = 3.317 \times 10^{27} \approx 3.3 \times 10^{27} \text{ W} \checkmark$ | 2 |
| c | i | <p>Peak wavelength at 420 nm✓</p> $T = \frac{2.9 \times 10^{-3}}{420 \times 10^{-9}} = 6905 \approx 6900 \text{ K} \checkmark$ | 2 |
| c | ii | $L = \sigma A T^4 = \sigma 4\pi R^2 T^4 \Rightarrow R = \sqrt{\frac{L}{\sigma 4\pi T^4}} \checkmark$ $R = \sqrt{\frac{3.317 \times 10^{27}}{5.67 \times 10^{-8} \times 4\pi \times 6905^4}} \checkmark$ $R = 1.4 \times 10^9 \text{ m} \checkmark$ | 3 |

18.

An HR diagram with the Sun marked is shown.



(a) Label

[4]

- (i) the main sequence by M,
- (ii) the region of white dwarfs by W,
- (iii) the region of red giants by R,
- (iv) the instability region by I.

(b) Arcturus has luminosity $170L_{\odot}$ and temperature 4300 K.

(i) Show that the radius of Arcturus is $24R_{\odot}$.

[2]

(ii) The mass of Arcturus is approximately the same as that of the Sun. Calculate the ratio of densities $\frac{\rho_A}{\rho_{\odot}}$.

[1]

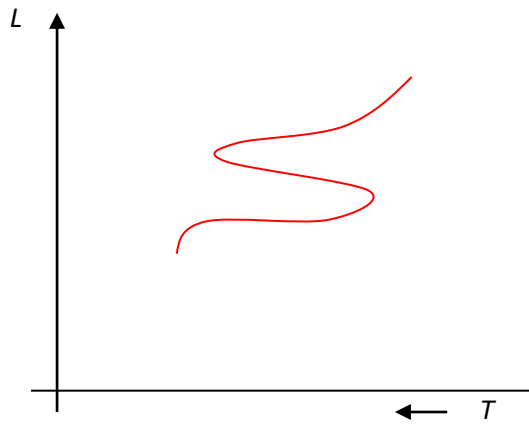
(iii) Label the approximate position of Arcturus on the HR diagram.

[1]

(iv) State the likely end stage in the evolution of Arcturus.

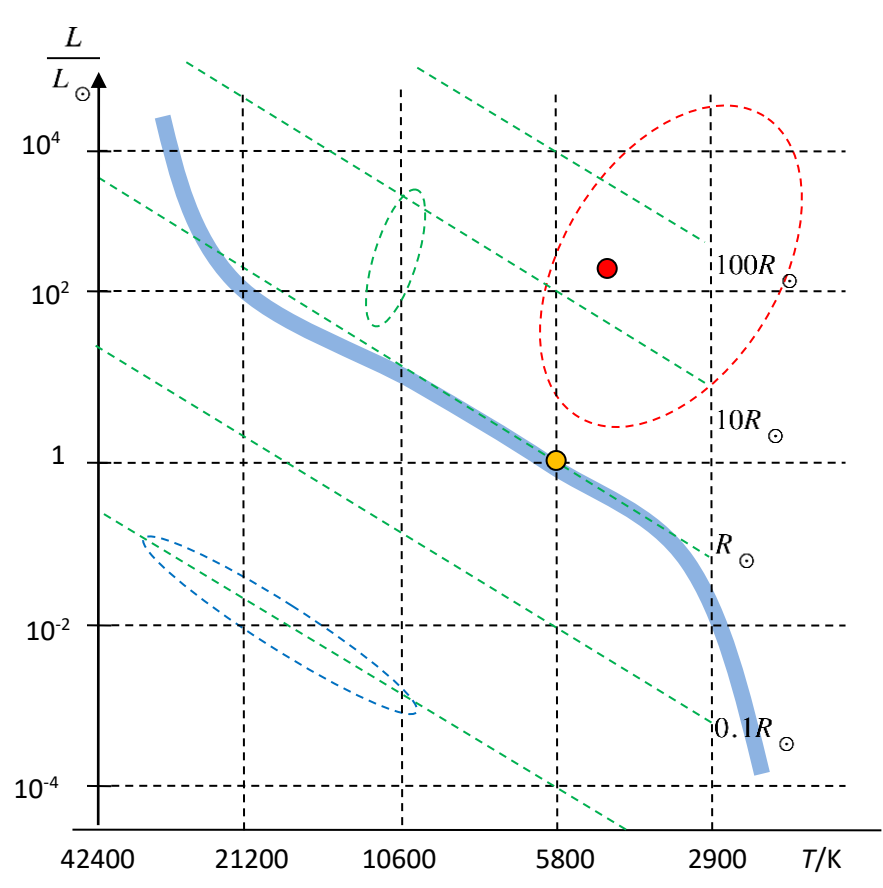
[1]

(c) The HR diagram shows the path of a star in the instability region.



Suggest the reason why the luminosity of the star is variable.

[2]

| Question 18 | | Answers | Marks |
|-------------|-----|---|-------|
| a | | <p>Blue thick line = Main sequence Red dotted line = Red giants Blue dotted line = White dwarfs Green dotted line = Instability region</p>  | 4 |
| b | i | $L = \sigma AT^4 = \sigma 4\pi R^2 T^4 \Rightarrow R = \sqrt{\frac{L}{\sigma 4\pi T^4}} \checkmark$ $\frac{R}{R_{\odot}} = \sqrt{\frac{L}{L_{\odot}}} \times \frac{5800^2}{4300^2} = \sqrt{170} \times \frac{5800^2}{4300^2} = 23.7 \checkmark$ | 2 |
| b | ii | $\frac{\rho_A}{\rho_{\odot}} = \left(\frac{R_{\odot}}{R}\right)^3 = \left(\frac{1}{23.7}\right)^3 = 7.5 \times 10^{-5} \checkmark$ | 1 |
| b | iii | See red dot in diagram in (a) ✓ | 1 |
| b | iv | White dwarf ✓ | 1 |
| c | | <p>The star is changing its size by pulsating ✓ And the temperature changes as well ✓ Leading to variations in the luminosity.</p> | 2 |

19.

(a) (i) Describe how a main sequence star maintains equilibrium.

[2]

(ii) State the common characteristic of main sequence stars.

[1]

(b) The star Vega has luminosity $40L_{\odot}$, temperature $1.6T_{\odot}$ and mass $2.1M_{\odot}$. Determine the ratios

(i) $\frac{R_V}{R_{\odot}}$ of radii,

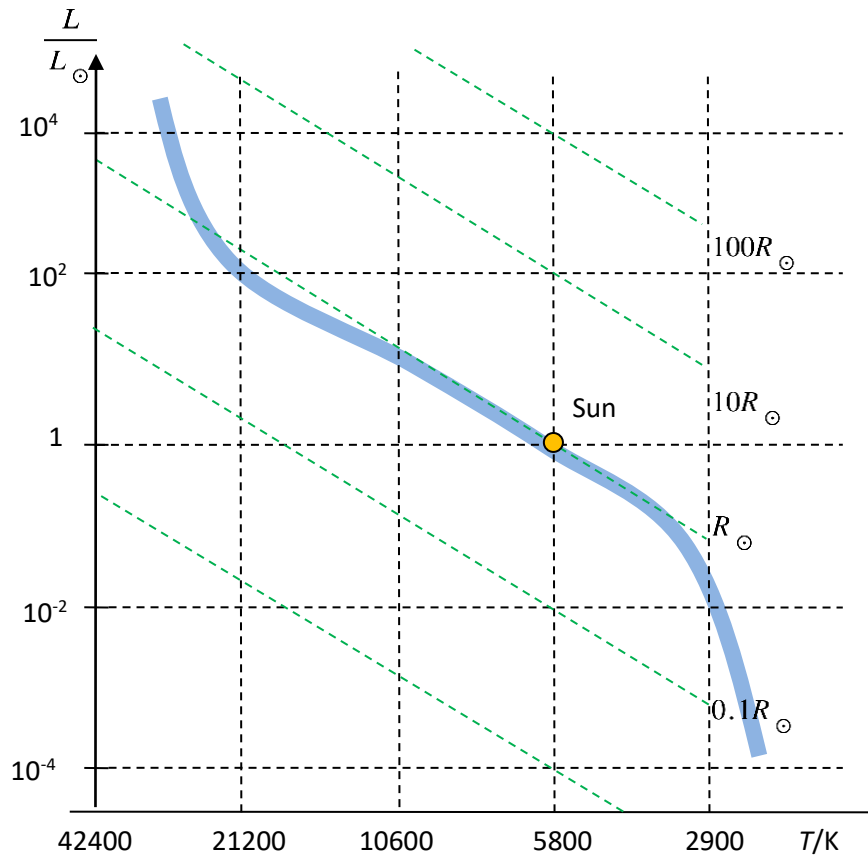
[2]

(ii) $\frac{\rho_V}{\rho_{\odot}}$ of densities.

[2]

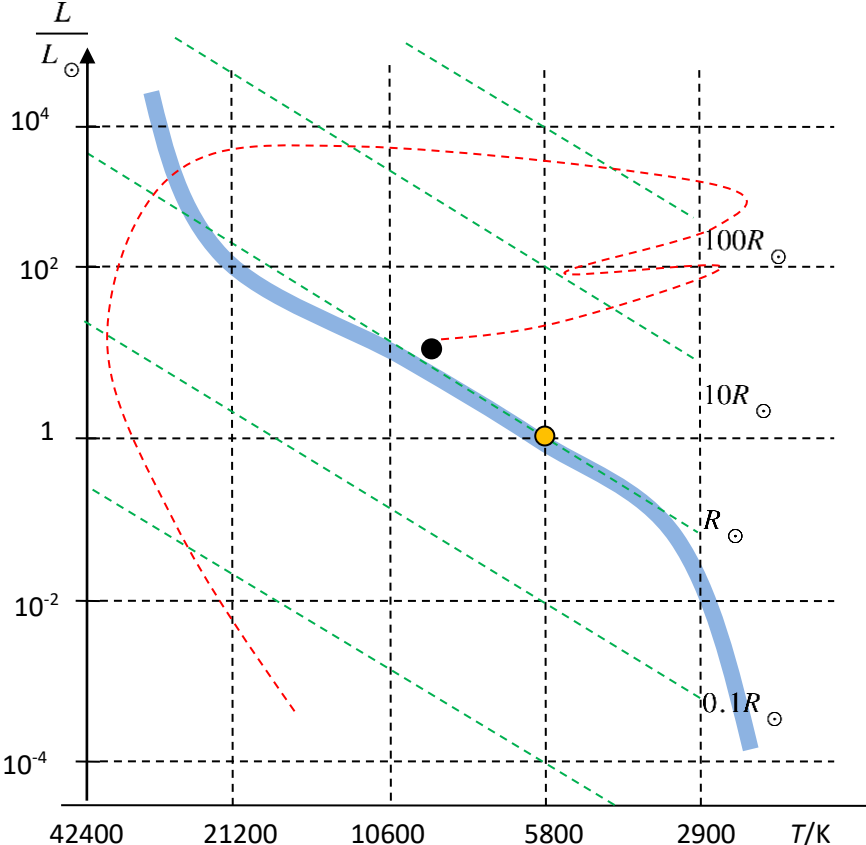
(c) Draw the expected evolutionary path of Vega on the HR diagram.

[3]



(d) Explain why the luminosity of Vega in the red giant phase will increase even though its surface temperature will decrease.

[2]

| Question 19 | | Answers | Marks |
|-------------|----|--|-------|
| a | i | There is a balance between the gravitational pressure that tends to shrink the star✓ And the outward radiation pressure generated by the energy produced in fusion in the core✓ | 2 |
| a | ii | They fuse hydrogen into helium✓ | 1 |
| b | i | $L = \sigma AT^4 = \sigma 4\pi R^2 T^4 \Rightarrow R = \sqrt{\frac{L}{\sigma 4\pi T^4}} \checkmark$ $\frac{R_V}{R_\odot} = \sqrt{\frac{L_V}{L_\odot} \left(\frac{T_\odot}{T_V}\right)^2} = \sqrt{40} \times \frac{1}{1.6^2} = 2.47 \approx 2.5 \checkmark$ | 2 |
| b | ii | $\frac{\rho_V}{\rho_\odot} = \frac{M_V}{M_\odot} \left(\frac{R_\odot}{R}\right)^3 \checkmark$ $\frac{\rho_V}{\rho_\odot} = (2.1) \times \left(\frac{1}{2.47}\right)^3 = 0.14 \checkmark$ | 2 |
| c | |  <p>Approximate position of Vega on the HR diagram ✓ Path towards the red giant region✓ then a counterclockwise curve from that region into the white dwarf region✓</p> | 3 |
| d | | Because the radius increases by a very large factor✓ And luminosity depends on the square of the radius✓ | 2 |

20.

The main sequence star Theta Carinae (θ Carinae) has mass $15M_{\odot}$ (of which 75% is hydrogen) and luminosity $2.6 \times 10^4 L_{\odot}$. (Solar mass = 2.0×10^{30} kg, solar luminosity = 3.8×10^{26} W.)

(a) Estimate

(i) the mass that gets converted into energy in one year. [3]

(ii) the time T it will take to convert 12% of the hydrogen mass of the star into energy. [2]

(b)

(i) It is suggested that for main sequence stars, the luminosity, L , is proportional to a power of the mass, M , i.e. $L \propto M^n$. Based on the data for θ Carinae, estimate that $n \approx 3.75$. [2]

(ii) Predict without any calculations whether the time T (defined in (a)(ii)) for the Sun will be larger or smaller than that for θ Carinae. [2]

(c) Suggest the likely sequence of events in the life of θ Carinae after the star leaves the main sequence. [3]

| Question 20 | | Answers | Marks |
|-------------|-----------|--|----------|
| a | i | Luminosity is $2.6 \times 10^4 \times 3.8 \times 10^{26} = 9.88 \times 10^{30} \text{ W}$ ✓ Energy released in 1 yr: $9.88 \times 10^{30} \times 365 \times 24 \times 60 \times 60 = 3.12 \times 10^{38} \text{ J}$ ✓ Equivalent to a mass: $\frac{3.12 \times 10^{38}}{(3 \times 10^8)^2} = 3.47 \times 10^{21} \text{ kg}$ ✓ | 3 |
| a | ii | Original mass of hydrogen: $0.12 \times 0.75 \times 15 \times 2.0 \times 10^{30} = 2.70 \times 10^{30} \text{ kg}$ ✓ $T = \frac{2.70 \times 10^{30}}{3.47 \times 10^{21}} = 7.8 \times 10^8 \text{ yr}$ ✓ | 2 |
| b | i | $15^n = 2.6 \times 10^4$ ✓ Taking logs: $n = 3.75$ ✓ | 2 |
| b | ii | θ Carinae fuses mass at a disproportionately higher rate ✓ And so, it takes much less time to fuse 12% of the available hydrogen mass compared to a lower mass star such as the Sun ✓ | 2 |
| c | | The star will leave the main sequence and move to the red supergiant region ✓ It will then explode as a supernova ✓ And will end up as a neutron star if the final core mass is below the Oppenheimer-Volkoff limit ✓ | 3 |